

# Hidden Orbital Order in $URu_2Si_2$

P. Chandra,<sup>1</sup> P. Coleman,<sup>2</sup> J. A. Mydosh<sup>3</sup> and V. Tripathi<sup>2</sup>

<sup>1</sup>*NEC, 4 Independence Way, Princeton, NJ 08540, U.S.A.*

<sup>2</sup>*Center for Materials Theory, Dept of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855, U.S.A.*

<sup>3</sup>*Kamerlingh Onnes Laboratory, Leiden University, P. O. Box 9504, 2300 RA Leiden, The Netherlands*

When matter is cooled from high temperatures, collective instabilities develop amongst its constituent particles that lead to new kinds of order.<sup>1</sup> An anomaly in the specific heat is a classic signature of this phenomenon. Usually the associated order is easily identified, but sometimes its nature remains elusive. The heavy fermion metal  $URu_2Si_2$  is one such example, where the order responsible for the sharp specific heat anomaly at  $T_0 = 17K$  has remained unidentified despite more than seventeen years of effort.<sup>2</sup> In  $URu_2Si_2$ , the coexistence of large electron-electron repulsion and antiferromagnetic fluctuations in  $URu_2Si_2$  leads to an almost incompressible heavy electron fluid, where anisotropically paired quasiparticle states are energetically favored.<sup>3</sup> In this paper we use these insights to develop a detailed proposal for the hidden order in  $URu_2Si_2$ . We show that incommensurate orbital antiferromagnetism, associated with circulating currents between the uranium ions, can account for the local fields and entropy loss observed at the  $17K$  transition; furthermore we make detailed predictions for neutron scattering measurements.

The intermetallic compound  $URu_2Si_2$  contains a dense lattice of local moments where quantum fluctuations barely prevent spin ordering; the residual antiferromagnetic couplings between the strongly repulsive electrons are large and can drive new types of collective instabilities.<sup>2</sup> At  $T_0 = 17.5K$ ,  $URu_2Si_2$  undergoes a second-order phase transition char-

acterized by sharp discontinuities in bulk properties including specific heat,<sup>4</sup> linear<sup>4</sup> and nonlinear<sup>5,6</sup> susceptibilities, thermal expansion<sup>7</sup> and resistivity.<sup>8</sup> The accompanying gap in the magnetic excitation spectrum<sup>9</sup> suggests the formation of an itinerant spin density wave at this temperature; however the size of the observed staggered moment<sup>10</sup> ( $m_0 = 0.03\mu_B$ ) *cannot* account for the entropy loss at this transition. The distinction between the primary hidden order parameter and the secondary magnetic order parameter is clarified by high-field measurements<sup>11,12</sup> which indicate that the bulk anomalies survive up to an applied field strength of  $40T$ , whereas the magnetically ordered moment is destroyed by fields less than half this magnitude ( $15T$ ).<sup>13</sup> Furthermore the size of the small ordered moment grows linearly with pressure,<sup>14</sup> while the gap associated with the hidden order is relatively robust over this pressure range.<sup>15</sup>

There have been many theoretical proposals for the primary order parameter in  $URu_2Si_2$ .<sup>16</sup> Two recent NMR studies have provided new insights on this outstanding problem with crucial consequences for the nature of the hidden order. It has been widely assumed that the spin antiferromagnetism and the hidden order are coupled and homogeneous. However a recent NMR study of  $URu_2Si_2$  under pressure<sup>17</sup> indicates that for  $T < T_0$  there exist *distinct* antiferromagnetic and paramagnetic regions, implying that the magnetic and the hidden orders are phase-separated. This conclusion, supported by  $\mu SR$  data,<sup>18</sup> implies that the hidden order phase contains no spin order. The observed growth of the staggered spin moment with applied pressure<sup>14</sup> is then simply a volume fraction effect which develops separately from the hidden order via a first order transition.<sup>19</sup> At ambient pressure roughly a tenth or less of the system<sup>17,18</sup> is magnetic with  $m_{spin} = 0.3\mu_B$ . The key implication of these measurements for theory is that the magnetic and the hidden order parameters are independent.<sup>19</sup>

In a parallel study, NMR measurements at ambient pressure<sup>20</sup> on  $URu_2Si_2$  indicate that at  $T \leq T_0$  the central (non-split) silicon NMR line-width develops a field-independent, isotropic component whose temperature-dependent magnitude is proportional to that of the hidden order parameter. These results imply an isotropic field distribution at the silicon

sites whose root-mean square value is proportional to the hidden order ( $\psi$ )

$$\langle B^\alpha(i)B^\beta(j) \rangle = A^2\psi^2\delta_{\alpha\beta}, \quad (1)$$

and is  $\sim 10$  Gauss at  $T = 0$ . This field magnitude is too small to be explained by the observed moment<sup>10</sup> which induces a field  $B_{spin} = \frac{8\pi}{3}\frac{M}{a^3} = 100$  Gauss where  $a$  is the  $U-U$  bond length ( $a = 4 \times 10^{-8}$  cm). Furthermore this moment is aligned along the  $c$ -axis, and thus cannot account for the isotropic nature of the local field distribution detected by NMR. These measurements indicate that as the hidden order develops, an isotropically distributed static magnetic field develops at each silicon site. This is *unambiguous* evidence that the hidden order parameter breaks time-reversal invariance.

Guided by these recent experiments, we now discuss our proposal for the nature of the hidden order parameter. The magnetic fields at the silicon nuclei have two possible origins:<sup>21</sup> the conduction electron-spin interaction and the orbital shift due to current densities. In  $URu_2Si_2$ , the electron fluid exhibits a strong Ising anisotropy along the  $c$ -axis, as measured by the Knight shift;<sup>20</sup> hence electron-spin coupling cannot be responsible for the observed isotropic fields. It would thus appear that these local fields are induced by currents that develop inside the crystal as the hidden order develops, and accordingly we attribute the observed isotropic linewidth to the orbital shift.

In this paper we propose that  $URu_2Si_2$  becomes an incommensurate orbital antiferromagnet (OAF) at  $T = T_0$  with charge currents circulating between the uranium ions. Similar states have been studied extensively in the context of the two-dimensional Hubbard model,<sup>22-26</sup> particularly in connection with staggered flux phases.<sup>27,28</sup> More recently commensurate current density wave order has been proposed to explain the spin-gap phase in the underdoped cuprate superconductors.<sup>29,30</sup> The planar tetragonal structure of  $URu_2Si_2$  lends itself naturally to an anisotropic charge instability of this type. Here we show that incommensurate orbital antiferromagnetic order in  $URu_2Si_2$  can quantitatively account for the existing specific heat and NMR data for  $T \leq T_0$ . In order to test this proposal, we make specific predictions for neutron scattering. Calculation of the structure factor re-

quires knowledge of the fields throughout the full volume in real-space. Since the NMR only yields this information at discrete points we need some additional input to proceed. In the orbital antiferromagnet, the spatial dependence of the fields throughout the sample is determined by the requirement that the field distribution at the silicon sites is isotropic. Using this approach, we are able to link quantitatively the fields observed by NMR to the large specific heat anomaly that develops at  $T_0$ . We also use the spatial field distribution associated with the incommensurate orbital antiferromagnet to predict the position, intensity and form-factor associated with neutron scattering peaks on a surface of constant anisotropy in momentum space centered on the origin.

We begin by estimating the local fields at the silicon sites due to orbital currents circulating around the square uranium plaquettes in the  $a - b$  plane. On dimensional grounds, the current along the  $U - U$  bond is given by  $I \sim \frac{e\Delta}{h}$  where  $\Delta$  is the gap associated with the formation of hidden order. Then the field induced at a height  $a$  above a plaquette is  $B \sim \left(\frac{2}{ac}\right) \left(\frac{e\Delta}{h}\right) = 11$  Gauss, in good agreement with the local field strength detected in *NMR* measurements; here<sup>4,9-11</sup> we have used  $\Delta = 110K$ . The resulting orbital moment,  $m_{orb} \sim 0.02\mu_B$ , is comparable to the effective spin moment at ambient pressure ( $m_0 = m_{spin}[P_{amb}] = [.10]m_{spin} = 0.03\mu_B$ ). We emphasize that an orbital moment produces a local field an order of magnitude less than that associated with a spin moment of the same value; the low field strengths observed at the silicon sites are quantitatively consistent with our proposal that they originate from charge currents.

An orbital moment,  $m_{orb} = 0.02\mu_B$ , can also account for the sizeable entropy loss at the transition. In a metal the change in entropy is given by  $\Delta S = \Delta\gamma_n T_0$ , where  $\Delta\gamma_n$  is the change in the linear specific heat coefficient resulting from the gapping of the Fermi surface.  $\Delta\gamma_n$  is inversely proportional to the Fermi energy  $\epsilon_F$  of the gapped Fermi surface. Thus in general, the change in entropy per unit cell is given by  $\Delta S \sim k_B \frac{k_B T_0}{\epsilon_F}$ . Exploiting the mean-field nature of this transition ( $2\Delta \propto T_0$ ), we find that

$$\Delta S_{OAFM} \sim k_B \left( \frac{m_{orb}}{\mu_B^*} \right) = 0.02 k_B \left( \frac{\mu_B}{\mu_B^*} \right) \quad (2)$$

where  $\mu_B^* = \left(\frac{e}{\hbar c}\right) a^2 \epsilon_F$ , is the saturated orbital moment reached when the hidden order gap  $\Delta$  is approximately  $\epsilon_F$ . The ratio  $\mu_B/\mu_B^* = (\frac{a_0}{a})^2 (\frac{\epsilon_H}{\epsilon_F})$ , where  $a_0$  is the Bohr radius and  $\epsilon_H = e^2/(2a_0)$  is the energy of a hydrogen atom. For  $URu_2Si_2$ , the very large size of  $\left(\frac{\epsilon_H}{\epsilon_F}\right) \sim 10^3$  produced by the large mass renormalization of the heavy electrons actually offsets the small ratio  $\frac{a_0}{a} \sim 10^{-1}$ , so that  $\mu_B/\mu_B^* \sim 10$ , and  $\Delta S_{OAFM} \sim 0.2k_B = 0.3k_B \ln 2$  is a number in good agreement with experiment.<sup>4</sup> The critical field for suppressing the associated thermodynamic anomalies is distinct from its spin counterpart; the ratio  $\frac{H_c^{orb}}{H_c^{spin}} \sim \frac{\mu_B}{\mu_B^*} \sim 10$  is qualitatively consistent with the observed critical field of approximately  $40T$  associated with the destruction of the hidden order<sup>11–13</sup>. From this simple discussion, we see that the sizable entropy loss associated with the development of the orbital antiferromagnetic state is a direct consequence of the strong renormalization of the electron mass in  $URu_2Si_2$  ( $\frac{m^*}{m} \propto \frac{\epsilon_H}{\epsilon_F}$ ). The absence of such a large effective mass ( $\frac{m^*}{m} \sim 3$ ) in the cuprates could explain why analogous thermodynamic anomalies are difficult to observe there.

Next we test whether the proposed orbital antiferromagnetism will yield the observed isotropic local fields. In order to do so, we allow the circulating current around a plaquette (cf. Fig. 1) centered at site  $\mathbf{X}$  to develop a modulated magnetization  $M(\mathbf{X}) = \psi e^{i\mathbf{Q} \cdot \mathbf{X}}$ . The current along a bond is then the difference of the circulating currents along its adjacent plaquettes. The field at a silicon site can be computed using Ampere's law, where the relevant vector potential is

$$\mathbf{A}(\mathbf{x}) = \frac{1}{c} \sum_j \int_{\mathbf{x}_j^{(1)}}^{\mathbf{x}_j^{(2)}} dx' \frac{\mathbf{I}(\mathbf{x}_j)}{|\mathbf{x} - (\mathbf{x}_j + \mathbf{x}')|}. \quad (3)$$

where  $\mathbf{x}_j^{(1,2)}$  are the endpoints of the bond at site  $\mathbf{x}_j$ .

The silicon atoms in  $URu_2Si_2$  are located at low-symmetry sites, so that the fields do not cancel; they reside above and below the centers of the uranium plaquettes. Microscopically, wave-vector selection is most likely due to details of the Fermi surface, however we can obtain a good idea of the likely modulation  $Q$  vector from the isotropic nature of the field distribution at the silicon sites. For example, in the commensurate case  $\mathbf{Q} = (1/2, 1/2, 1)$ , the fields on the silicon sites are only along the c-axis and thus the field distribution would

be highly anisotropic. Consider a wave-vector  $(q, q, 1)$  (c.f. Fig. 1). In this case, the currents are modulated within a plane with a wavelength  $2\pi/q$ , but staggered between planes. This then produces a circulating field distribution where the component of the field parallel to the  $(0, 0, 1)$  planes becomes larger and larger as  $q$  is reduced. To obtain an isotropic field distribution,  $q$  needs to be reduced to a point where the horizontal and vertical components of the field are comparable. A detailed calculation based on the above model, shows that the condition of perfect isotropy yields a circle of wave-vectors (Fig. 2.) centered around  $Q = (0, 0, 1)$  with a radius  $q \approx 0.22$ . Relaxation of this constraint results in an annulus of possible  $Q$  vectors as shown in Fig. 2.

Our proposal of incommensurate current ordering in  $URu_2Si_2$  can be tested by experiment. In particular we can Fourier transform the real-space magnetic fields to calculate the neutron scattering cross-section

$$\frac{d\sigma}{d\Omega} = \left( \frac{g_N e}{8\pi\hbar c} \right)^2 |\vec{B}(q)|^2 = r_o^2 S(q). \quad (4)$$

Here  $|\vec{B}(q)|^2$  is the structure factor of the magnetic fields produced by the orbital currents and  $S(q) = |B(q)|^2 / (4\pi\mu_B)^2$  is the structure factor of the orbital magnetic moments, measured in units of electron Bohr magnetons ( $\mu_B$ ). The factor  $r_o = \frac{g_N e^2}{4m_e c^2}$ ,  $m_e$  is the electron mass and  $g_N$  is the neutron gyromagnetic ratio. Using the vector potential from eq. (3), we have calculated the magnetic field distribution for the *incommensurate* orbital antiferromagnet described above, and find that its Fourier transform is given by

$$\vec{B}(q) = \frac{4\pi}{c} N I a^2 \sum_{\mathbf{G}_{n_1, n_2, n_3}} \delta_{q, Q+G} j_0 \left[ \frac{q_x a}{2} \right] j_0 \left[ \frac{q_y a}{2} \right] (1 + (-1)^{(n_1+n_2+n_3)}) \cdot \left( \frac{q_y \hat{x} - q_x \hat{y}}{2q} \right) \times \hat{q}, \quad (5)$$

where  $j_0(x) = \frac{\sin x}{x}$ . From this expression, we can determine the intensities and the form factors associated with the diffraction. We find that there exists a set of dominant peaks associated with a constant anisotropy locus of wavevectors (Fig. 2.) in the first Brillouin zone (BZ) where  $S(q) = 0.18 \left( \frac{N I a^2}{c \mu_B} \right)^2$ , indicating that roughly a fifth of the total integrated weight of  $S(q)$  (TIW) lies here. Using the sum rule that relates the TIW to the square of the moment, and the fact that at ambient pressure only a tenth of the sample is (spin)

magnetic, we find that the scattering peaks due to orbital ordering in the first BZ should have 1/50 the intensity of the analogous spin magnetic peaks at ambient pressure. We have also calculated that these peaks will have a form-factor (cf. inset Fig. 2) that scales with wavevector as  $q^{-4}$ , where this power-law decay is signatory of an extended scattering source.

We end with remarks about the microscopic formation of these charge currents. At low temperatures heavy electron materials form almost incompressible Fermi liquids. The large Coulomb repulsion between the Landau quasiparticles strongly suppresses on-site charge fluctuations, demanding nodes in the particle-hole wavefunction. The resulting anisotropically paired states are also favored by the residual antiferromagnetic interactions that persist in the heavy electron fluid. Indeed we believe that the same d-wave pairing that drives the superconducting transition in  $URu_2Si_2$  at  $T = 1.5K$  also plays an important role in the formation of the orbital antiferromagnetism. We have found that the development of such anisotropic charge-density wave pairing occurs naturally in a simple model of  $URu_2Si_2$  with nearest-neighbor antiferromagnetic interactions,

$$H_I = \sum_{\vec{q}} J(\vec{q}) \vec{S}(\vec{q}) \cdot \vec{S}(-\vec{q}). \quad (6)$$

Here  $\vec{S}(\vec{q})$  is the Fourier transform of the magnetization at wavevector  $\vec{q}$  and  $J(\vec{q}) = 2J(\cos q_x + \cos q_y)$  for a nearest-neighbour interaction between adjacent  $U$  atoms in the basal plane. Expanding this interaction in terms of quasiparticle operators, we find that it can be re-written as a sum of attractive interactions in four independent anisotropic charge density wave channels:

$$H_I = -J \sum_{\Gamma=1,4,k_1,k_2,Q} (\gamma_{k_1}^\Gamma)^* \gamma_{k_2}^\Gamma \rho_{k_1}(Q) \rho_{k_2}(-Q). \quad (7)$$

Here  $\rho_k(Q) = \sum_{\sigma=\pm 1/2} c_{k-Q/2\sigma}^\dagger c_{k+Q/2\sigma}$  is the charge density operator at wavevector  $Q$ , expressed in terms of creation and annihilation operators of the heavy quasiparticles. The form factors are  $\gamma^{1,2}(\vec{k}) = \cos(k_x) \pm \cos(k_y)$ ,  $\gamma^{3,4}(\vec{k}) = i(\sin(k_x) \pm \sin(k_y))$ . Of these four possibilities, channels 1 and 3 do not have nodes, and are therefore suppressed by local Coulomb interactions. In the remaining two channels, only  $\Gamma = 4$  breaks time reversal

symmetry, giving rise to incommensurate orbital currents. One of the questions for further study concerns the excitations of this mean-field state. In particular, an incommensurate OAF is expected to exhibit a gapless phason mode associated with slow fluctuations of its wavevector.  $URu_2Si_2$  is known to develop a dispersing singlet excitation<sup>10</sup> at  $T = T_0$ . This mode was previously attributed to spin antiferromagnetism, now known to be absent from the hidden order phase.<sup>17</sup> We plan to study the possible identification of this propagating mode with the phason of an incommensurate orbital antiferromagnet.

In summary, we have discussed the theoretical implications of two recent NMR experiments on the hidden order in  $URu_2Si_2$ . Pressure-dependent measurements indicate that it is completely independent of the spin magnetism in this material. We argue that the development of isotropically distributed magnetic fields at the silicon sites at  $T = T_0$  implies that the hidden order parameter breaks time-reversal symmetry. The size and the isotropy of these observed local fields lead us to propose that  $URu_2Si_2$  becomes an incommensurate orbital antiferromagnet at  $T < T_0$ . The heavy electron mass reduces the saturation value of the orbital moment, accounting for the sizable entropy loss at the transition and the scale of the associated critical field. We calculate the positions, intensities and form-factor associated with the resulting neutron scattering peaks so that this proposal can be tested by experiment.



## REFERENCES

- <sup>1</sup> Goodstein D.L., *States of Matter* (Dover, New York, 1985).
- <sup>2</sup> Buyers W.J.L., “Low Moments in Heavy-Fermion Systems,” *Physica B* **223 & 224**, 9-14 (1996).
- <sup>3</sup> Miyake K. , Schmitt-Rink S. and Varma C.M., “Spin-Fluctuation-Mediated Even-Parity Pairing in Heavy Fermion Superconductors,” *Phys. Rev. B* **34**, 6554-6556 (1986).
- <sup>4</sup> Palstra T.T.M. et al., “Superconducting and Magnetic Transitions in the Heavy Fermion System  $URu_2Si_2$ ,” *Phys. Rev. Lett.* **55**, 2727-2730 (1985).
- <sup>5</sup> Miyako Y. et al., “Magnetic Properties of  $U(Ru_{1-x}Rh_x)_2Si_2$  Single Crystals ( $0 < x < 1$ ),” *J. Appl. Phys.* **70**, 5791-5793 (1991).
- <sup>6</sup> Ramirez A.P. et al., “Nonlinear Susceptibility as a Probe of Tensor Spin Order in  $URu_2Si_2$ ,” *Phys. Rev. Lett.* **68**, 2680-2683 (1992).
- <sup>7</sup> De Visser A. et al., “Thermal Expansion and Specific Heat of Monocrystalline  $URu_2Si_2$ ,” *Phys. Rev. B* **34**, 8168-8171 (1986).
- <sup>8</sup> Palstra T.T.M., Menvosky A.A. and Mydosh J.A., “Anisotropic Electrical Resistivity of the Magnetic Heavy-Fermion Superconductor  $URu_2Si_2$ ,” *Phys. Rev. B* **33**, 6527-6530 (1986).
- <sup>9</sup> Mason T.E. and Buyers W.J.L., “Spin Excitations and the Electronic Specific Heat of  $URu_2Si_2$ ,” *Phys. Rev. B* **43**, 11471-11473 (1991).
- <sup>10</sup> Broholm C. et al., “Magnetic Excitations in the Heavy-Fermion Superconductor  $URu_2Si_2$ ,” *Phys. Rev. B* **43**, 12809-12822 (1991).
- <sup>11</sup> Mentink S.A.M. et al., “Gap Formation and Magnetic Ordering in  $URu_2Si_2$  probed by High-Field Magnetoresistance,” *Phys. Rev. B* **53**, R6014-R6017 (1996).

- <sup>12</sup> van Dijk N.H. et al., “Specific Heat of Heavy-Fermion  $URu_2Si_2$  in High Magnetic Fields,” *Phys. Rev. B* **56**, 14493-14498 (1997).
- <sup>13</sup> Mason T.E. et al., “Nontrivial Magnetic Order in  $URu_2Si_2$ ?” *J. Phys.: Condens. Matt.* **7**, 5089-5096 (1995).
- <sup>14</sup> Amitsuka H. et al., “Effect of Pressure on Tiny Antiferromagnetic Moment in the Heavy-Electron Compound  $URu_2Si_2$ ,” *Phys. Rev. Lett.* **83**, 5114-5117 (1999).
- <sup>15</sup> Fisher, R.A. et al., “Specific Heat of  $URu_2Si_2$ : Effect of Pressure and Magnetic Field on the Magnetic and Superconducting Transitions,” *Physica B* **163**, 419-423 (1990).
- <sup>16</sup> Shah N., Chandra P., Coleman P., and Mydosh J.A., “Hidden Order in  $URu_2Si_2$ ,” *Phys. Rev. B* **61**, 564-569 (2000).
- <sup>17</sup> Matsuda K. et al., “Spatially Inhomogeneous Development of Antiferromagnetism in  $URu_2Si_2$ : Evidence from  $^{29}Si$  NMR Under Pressure”, *Phys. Rev. Lett.* **87**, 087203-1–087203-4 (2001).
- <sup>18</sup> Luke G.M. et al., “Muon Spin Relaxation in Heavy Fermion Systems,” *Hyperfine Inter.* **85**, 397-409 (1994).
- <sup>19</sup> Chandra P., Coleman P. and Mydosh J.A., “Pressure-Induced Magnetism and Hidden Order in  $URu_2Si_2$ ,” cond-mat/0110048.
- <sup>20</sup> Bernal O.O. et al., “ $^{29}Si$  NMR and Hidden Order in  $URu_2Si_2$ ,” *Phys. Rev. Lett.* **87**, 153-156 (2001).
- <sup>21</sup> Schlichter C.P., *Principles of Magnetic Resonance*, (Springer-Verlag, Berlin 1978).
- <sup>22</sup> Halperin B.I. and Rice T.M., *Solid State Phys.* **21**, eds. by F. Seitz, D. Turnbull and H. Ehrenreich, (Academic Press, New York, 1968).
- <sup>23</sup> Affleck I. and Marston J.B., “Large-n Limit of the Heisenberg-Hubbard Model: Implications for High  $T_c$  superconductors,” *Phys. Rev. B* **37**, 3774-3777 (1988).

- <sup>24</sup> Kotliar G., “Resonating Valence Bonds and D-Wave Superconductivity,” *Phys. Rev. B* **37**, 3664-3666 (1988).
- <sup>25</sup> Nersesyan A.A. and Vachnadze G.E., “Low-Temperature Thermodynamics of Two-Dimensional Orbital Antiferromagnet,” *J. Low Temp. Phys.* **77**, 293-303 (1989).
- <sup>26</sup> Schulz H.J., “Fermi Surface Instabilities of a Generalized Two-Dimensional Hubbard Model,” *Phys. Rev. B* **39**, 2940-2943 (1989).
- <sup>27</sup> Hsu T.C., Marston J.B. and Affleck I., “Two Observable Features of the Staggered-Flux Phase at Nonzero Doping,” *Phys. Rev. B* **43**, 2866-2877 (1991).
- <sup>28</sup> P.A. Lee, “Orbital Currents in Underdoped Cuprates,” cond-mat/0201052.
- <sup>29</sup> Chakravarty S., Laughlin R.B., Morr D.K. and Nayak C., “Hidden Order in the Cuprates,” *Phys. Rev. B* **63**, 094503-1–094503-10 (2001).
- <sup>30</sup> Kee H.-Y. and Kim Y.B., “Specific Heat Anomaly in the D-Density Wave State and Emergence of Incommensurate Orbital Antiferromagnetism,” cond-mat/0111461.

We would like to acknowledge discussions with G. Aeppli, H. Amitsuka, O. Bernal, S. Chakravarty, L.P. Gorkov, G. Lonzarich, K. McEuen, D. McLaughlin, D. Morr and C. Nayak. P. Coleman and V. Tripathi are supported by the National Science Foundation.

## Figure Captions

**Fig. 1.** Magnetic field distribution associated with incommensurate orbital currents in the  $(0,0,1)$  plane. (a) Schematic illustration of incommensurate orbital currents, showing resulting magnetic fields above and below the  $(0,0,1)$  plane, (b) Side view showing how proposed field distribution is isotropic at silicon sites with a staggered current distribution between layers corresponding to  $Q = (0.16, 0.16, 1)$ .

**Fig. 2.** An isotropic magnetic field distribution at the silicon sites can be produced by many different wave vectors  $Q$  for orbital order, all of which give qualitatively similar neutron scattering patterns. (a) Contour plot showing locus of constant anisotropy around a ring of radius  $Q_{\perp} = 0.22$  in the vicinity of  $Q = (0,0,1)$ . (b) Predicted elastic neutron scattering intensity, where  $\tilde{S}(q)^{\frac{1}{2}} = S(q)^{\frac{1}{2}} / \left( \frac{NIa^2}{c\mu_B} \right) \sim \sqrt{|B(q)|^2}$  gives a measure of the Fourier spectrum of magnetic fields  $B(q)$ , plotted for the representative case  $Q = (0.16, 0.16, 1)$ .

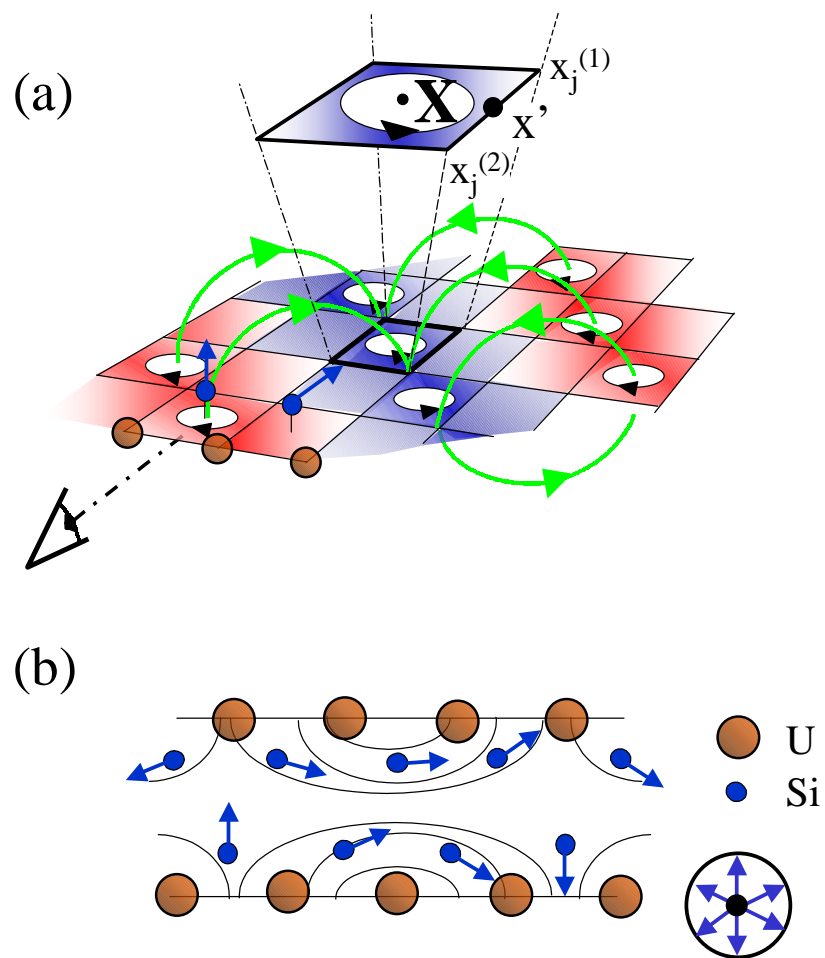


Fig. 1

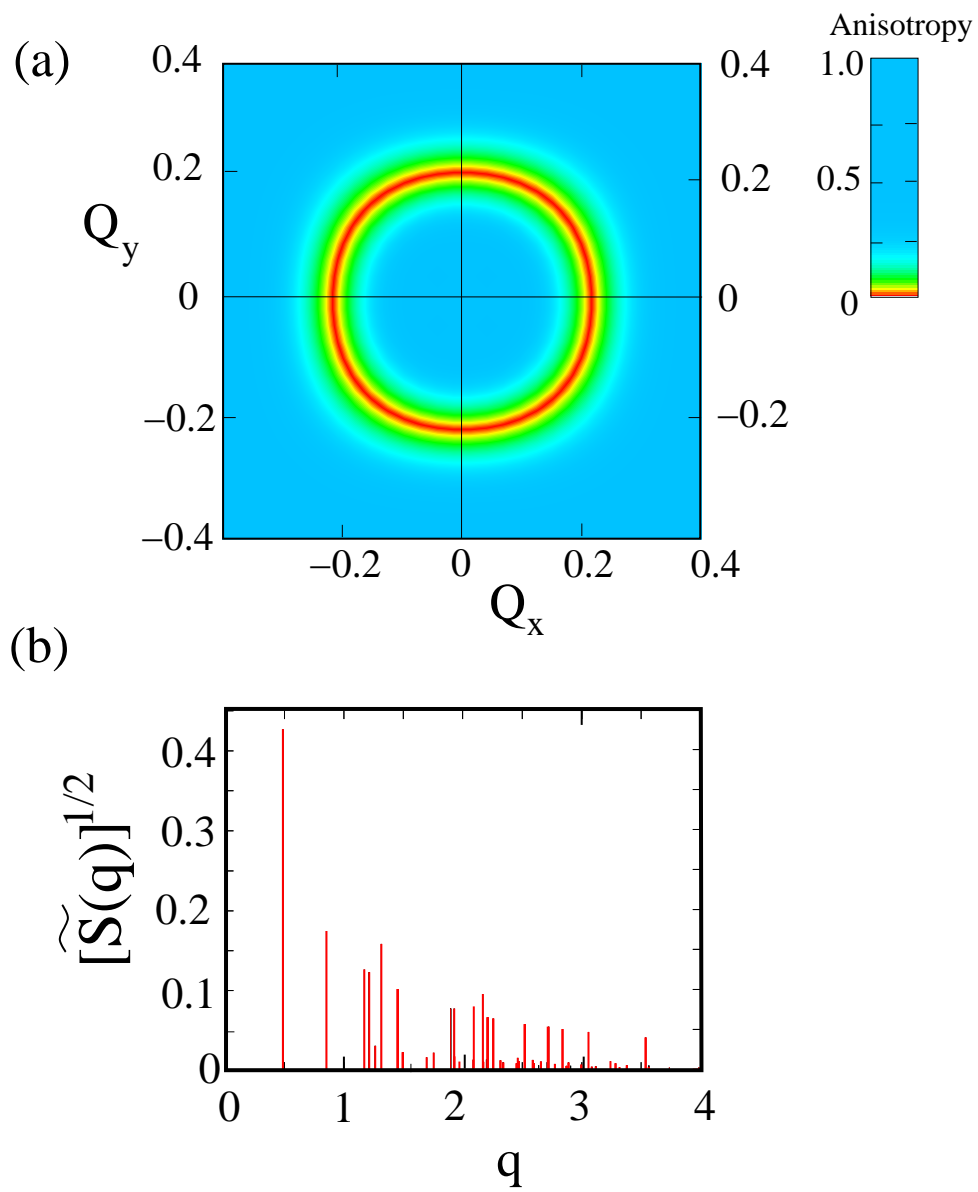


Fig. 2